PΣΛK
PHYSICS & CHEMISTRY

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Module 2: Dynamics | Topic 2.2: Force Analysis

Solutions

Foundation

1. Describe the difference between static friction and kinetic friction, and state the formulae used to calculate each type of friction.

   Static friction ($f_s = \mu_s N$) is experienced by a stationary object and this needs to be overcome before the object can begin moving. Once the object has started moving, it experiences kinetic friction ($f_k = \mu_k N$).

2. (a) Calculate the normal force acting on a 4 kg mass that is at rest on a horizontal surface.

$$\begin{aligned}\Sigma F_y = N - mg &= 0\\ N &= mg\\ &= 4\text{ kg} \times 9.8\text{ m s}^{-2}\\ &= 39.2\text{ N}\end{aligned}$$

   (b) Calculate the maximum static frictional force experienced by the mass if the coefficient of static friction is 0.74.

$$\begin{aligned}f_s &= \mu_s N\\ &= 0.74 \times 39.2\text{ N}\\ &= 29.01\text{ N}\end{aligned}$$

   (c) Calculate the kinetic frictional force experienced by the mass if the coefficient of static friction is 0.56.

$$\begin{aligned}f_k &= \mu_k N\\ &= 0.56 \times 39.2\text{ N}\\ &= 21.95\text{ N}\end{aligned}$$

   (d) A force of 30 N is now applied horizontally to the same 4 kg mass.

   Calculate the acceleration of the block when it is moving.

$$\begin{aligned}\Sigma F_x = 30\text{ N} - f_k &= ma\\ \implies a &= \frac{30\text{ N} - f_k}{m}\\ &= \frac{30\text{ N} - 21.95\text{ N}}{4\text{ kg}}\\ &= 2.01\text{ m s}^{-2}\end{aligned}$$

Development

1. A 20 kg block of ice is pushed along a rough surface with a horizontal force of 60 N. The coefficient of kinetic friction between the ice and the surface is 0.25.

   What is the net horizontal force acting on the block of ice when it is moving?

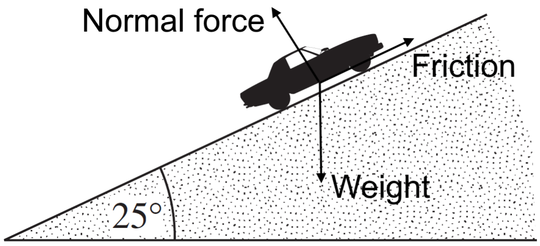
   (a) 11 N
   (b) 146 N
   (c) 196 N
   (d) 246 N

2. A bicycle rider exerts a forwards force of 400 N through the pedals of the bike when riding on a horizontal surface. The total mass of the rider and the bike is 85 kg, and their acceleration is 2 $\mathrm{m\,s^{-2}}$.

   From this information, it can be concluded that:

   (a) There is no frictional force acting on the bike at this time
   (b) Newton's 2^{nd} law of motion is not applicable to this motion
   (c) The coefficient of static friction between the bike and the surface is 0.28
   (d) The coefficient of kinetic friction between the bike and the surface is 0.28

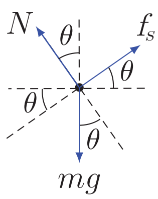
3. A car of mass 1540 kg is stationary on a hill that has a slope of 25°.



   (a) On the diagram, draw and label all the forces acting on the car. **2**

   2 marks – Draws and labels the forces acting on the car including the direction

   (b) Calculate the force that is necessary to stop the car from moving down the slope. **2**

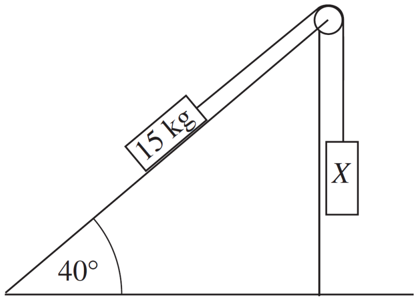


   Resolving forces ($\nearrow^{+}$),

$$\Sigma F_x = f_s - mg\sin\theta = 0$$
$$\implies f_s = mg\sin\theta$$
$$= 1540\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 25°$$
$$= 6378.2\text{ N}$$

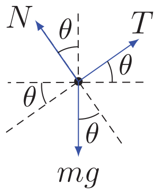
2 marks – Calculates the correct force required

4. A 15 kg block is placed on a smooth inclined plane. It is attached by a light, inextensible string over a frictionless pulley to block X.



(a) The 15 kg mass accelerates down the slope at 0.50 m s^{-2}.

i. Calculate the tension in the string. **2**

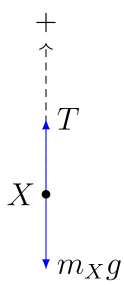


Resolving forces (+ ↙),

$$\Sigma F_x = -T + mg\sin\theta = ma$$
$$\implies T = mg\sin\theta - ma$$
$$= 15 \text{ kg} \times 9.8 \text{ m s}^{-2} \times \sin 40° - 15 \text{ kg} \times 0.50 \text{ m s}^{-2}$$
$$= 87 \text{ N}$$

2 marks – Resolves forces and calculates the correct tension

ii. Calculate the mass of block X. **2**



$$\Sigma F_y = T - m_X g = m_X a$$
$$T = m_X g + m_X a$$
$$= m_X(g + a)$$

$$
\begin{aligned}
m_X &= \frac{T}{g+a} \\
&= \frac{87\ \text{N}}{9.8\ \text{m s}^{-2} + 0.50\ \text{m s}^{-2}} \\
&= 8.4\ \text{kg}
\end{aligned}
$$

2 marks – Sums up forces and calculates the correct tension

(b) The mass of block X is changed so that the resultant force acting on the 15 kg mass is zero. **2**

Calculate the new mass of block X.

For the 15 kg block,

$$
\begin{aligned}
\Sigma F_x = -T + mg\sin\theta &= 0 \\
T &= mg\sin\theta \\
&= 15\ \text{kg} \times 9.8\ \text{m s}^{-2} \times \sin 40^\circ \\
&= 94.5\ \text{N}
\end{aligned}
$$

For block X,

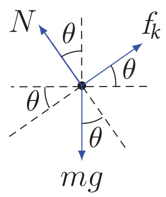
$$
\begin{aligned}
\Sigma F_y = T - m_X g &= 0 \\
m_X &= \frac{T}{g} \\
&= \frac{94.5\ \text{N}}{9.8\ \text{m s}^{-2}} \\
&= 9.6\ \text{kg}
\end{aligned}
$$

1 mark – Calculates the new tension force

1 mark – Calculates the new mass of X

5. A girl of mass 40.0 kg slides at a constant speed down a dip inclined at 22° to the horizontal.

(a) Calculate the frictional force acting on the girl and the coefficient of kinetic friction between the girl and the surface of the dip. **3**



Resolving forces ($^{+}\nwarrow$ and $_{+}\swarrow$),

$$\Sigma F_y = N - mg\cos\theta = 0$$
$$N = mg\cos\theta$$
$$= 40.0\ \text{kg} \times 9.8\ \text{m s}^{-2} \times \cos 22°$$
$$= 363.46\ \text{N}$$

$$\Sigma F_x = -f_k + mg\sin\theta = 0$$
$$\implies f_k = mg\sin\theta$$
$$= 40.0\ \text{kg} \times 9.8\ \text{m s}^{-2} \times \sin 22°$$
$$= 146.85\ \text{N}$$
$$f_k = \mu_k N$$
$$\implies \mu_k = \frac{f_k}{N}$$
$$= \frac{146.85\ \text{N}}{363.46\ \text{N}}$$
$$= 0.40$$

1 mark – Calculates the correct normal force

1 mark – Calculates the correct frictional force

1 mark – Calculates the correct coefficient of friction

(b) Suppose the girl accelerated down the incline at 1.84 $m\,s^{-2}$ instead of moving at constant speed because of a decrease in friction. **2**

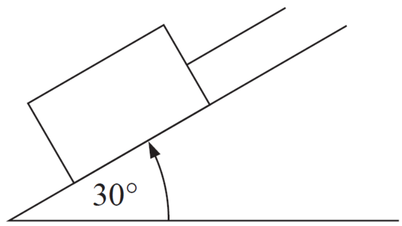
Calculate the magnitude of the change in the frictional force.

$$\Sigma F_x = -f_k + mg\sin\theta = ma$$
$$\implies f_k = mg\sin\theta - ma$$
$$= 40.0\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 22^\circ - 40.0\text{ kg} \times 1.84\text{ m s}^{-2}$$
$$= 73.25\text{ N}$$
$$|\Delta f_k| = |73.25\text{ N} - 146.85\text{ N}|$$
$$= 73.6\text{ N}$$

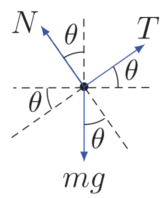
1 mark – Calculates the new frictional force

1 mark – Calculates the magnitude of the change in friction

6. A 3.7 kg box is initially held stationary on a smooth slope at 30° by a lightweight inextensible string. Ignore the effects of friction.



(a) Calculate the tension in the string when the box accelerates up the slope with a magnitude of 1.7 $m\,s^{-2}$. **2**



Resolving forces ($\nearrow^+$),

$$\Sigma F_x = T - mg\sin\theta = ma$$
$$\implies T = mg\sin\theta + ma$$
$$= 3.7\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 30^\circ + 3.7\text{ kg} \times 1.7\text{ m s}^{-2}$$
$$= 24.42\text{ N}$$

2 marks – Resolves forces and calculates the correct tension

(b) Describe the motion of the box when the tension in the string is 16 N. Include relevant calculations to support your answer. **2**

$$\begin{aligned} \Sigma F_x &= T - mg\sin\theta \\ &= 16\text{ N} - 3.7\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 30^\circ \\ &= -2.13\text{ N} \\ \Sigma F_x &= ma \\ \implies a &= \frac{\Sigma F_x}{m} \\ &= \frac{-2.13\text{ N}}{3.7\text{ kg}} \\ &= -0.58\text{ m s}^{-2} \end{aligned}$$

The box will slide down the slope with an acceleration of 0.58 m s^{-2} down the slope.

1 mark – Describes the motion of the block

1 mark – Includes a relevant calculation of the net force and/or acceleration

The tension in the string and the angle of the slope can be changed in order to observe the motion of the box under different conditions.

(c) The box accelerates up the slope when the tension in the string is 35 N and the slope is at 30°. **2**

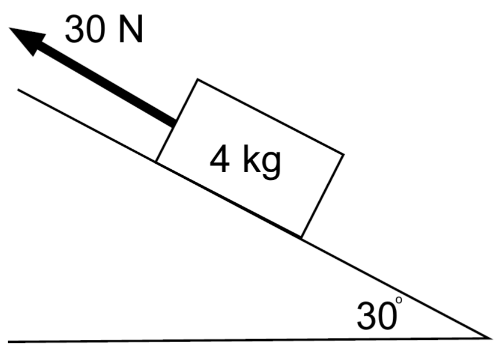
If the slope is changed to 50°, should the tension in the string be smaller or larger to keep the same acceleration? Explain your answer.

The net force acting along the slope is given by: $\Sigma F_x = T - mg\sin\theta = ma$. Increasing the angle of the slope (θ) would increase the component of the weight force acting down the slope ($mg\sin\theta$). As such, the tension in the string (T) must be larger so that the net force and acceleration can be kept the same.

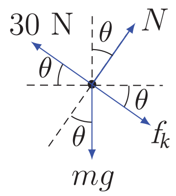
1 mark – Predicts that the tension must be larger

1 mark – Explains why in terms of the component of the weight force acting down the slope

7. A 4 kg block is pulled up an inclined plane at a constant velocity with a force of 30 N as shown in the diagram below.



(a) Calculate the kinetic frictional force and the coefficient of kinetic friction between the block and the inclined plane. **3**



Resolving forces ($\nearrow^+$ and $^+\nwarrow$),

$$\Sigma F_y = N - mg\cos\theta = 0$$
$$N = mg\cos\theta$$
$$= 4\text{ kg} \times 9.8\text{ m s}^{-2} \times \cos 30°$$
$$= 33.95\text{ N}$$

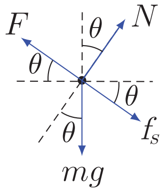
$$\Sigma F_x = 30\text{ N} - f_k - mg\sin\theta = 0$$
$$\implies f_k = 30\text{ N} - mg\sin\theta$$
$$= 30\text{ N} - 4\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 30°$$
$$= 10.4\text{ N}$$
$$f_k = \mu_k N$$
$$\implies \mu_k = \frac{f_k}{N}$$
$$= \frac{10.4\text{ N}}{33.95\text{ N}}$$
$$= 0.31$$

1 mark – Calculates the correct normal force

1 mark – Calculates the correct frictional force

1 mark – Calculates the correct coefficient of friction

(b) If the coefficient of static friction is two times that of the coefficient of kinetic friction, calculate the minimum pulling force required to move the block if it is initially stationary. **2**

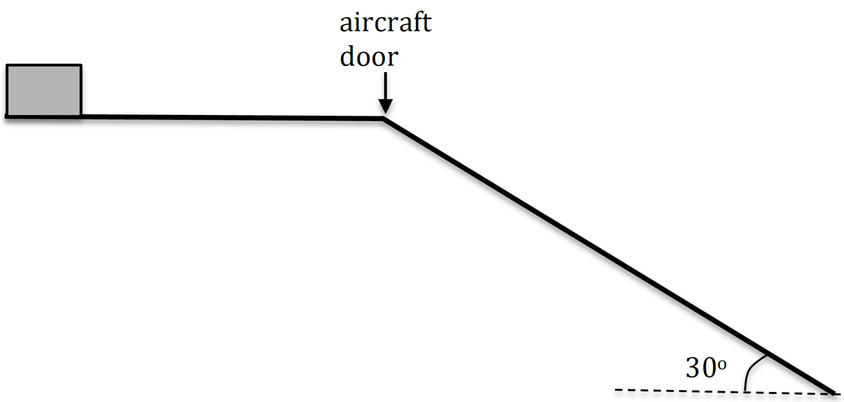


Resolving forces (+ ↖),

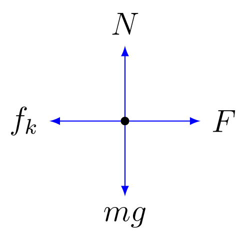
$$
\begin{aligned}
\Sigma F_x = F - f_s - mg\sin\theta > 0 \\
\implies F &> f_s + mg\sin\theta \\
&= \mu_s N + mg\sin\theta \\
&= 2 \times 0.31 \times 33.95\ \text{N} + 4\ \text{kg} \times 9.8\ \text{m s}^{-2} \times \sin 30^\circ \\
&= 40.4\ \text{N}
\end{aligned}
$$

2 marks – Calculates the correct minimum pulling force required

8. A rectangular crate of mass 35 kg is being unloaded from an aeroplane. Inside the aeroplane, it is pushed across a rough horizontal floor to the door of the aircraft at a constant speed of $1\ \mathrm{m\,s^{-1}}$. The coefficient of kinetic friction between the crate and the floor of the aircraft is 0.45.



(a) Draw a free body diagram showing all the forces acting on the crate while it moves across the horizontal floor. **1**



1 mark – Draws the correct free body diagram

(b) What is the acceleration of the crate while the crate is being pushed across the horizontal floor of the aircraft? **1**

0 (the block is moving at constant speed)

1 mark – Identifies the correct acceleration

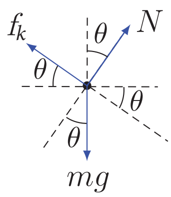
(c) Calculate the horizontal force that is pushing the crate across the floor of the aircraft. **2**

$$\begin{aligned}
\Sigma F_y = N - mg &= 0 \\
N &= mg \\
&= 35\ \mathrm{kg} \times 9.8\ \mathrm{m\,s^{-2}} \\
&= 343\ \mathrm{N} \\
\Sigma F_x = F - f_k &= 0 \\
F &= f_k \\
&= \mu_k N \\
&= 343\ \mathrm{N} \times 0.45 \\
&= 154.35\ \mathrm{N}
\end{aligned}$$

2 marks – Calculates the correct force

(d) The crate is pushed out of the aircraft door and allowed to slide down a rough ramp that makes an angle of 30° to the horizontal. The coefficient of kinetic friction between the crate and the ramp is 0.40.

i. Draw a free body diagram showing all the forces acting on the crate while it is on the ramp. **1**



1 mark – Draws the correct free body diagram

ii. Calculate the magnitude of the acceleration of the crate while it is sliding down the ramp. **3**

Resolving forces ($\nearrow^{+}$ and $\searrow_{+}$),

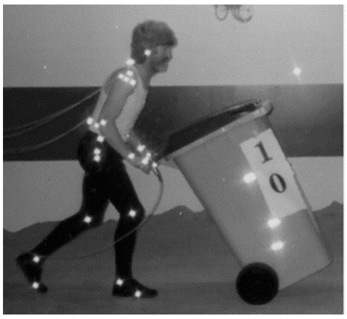
$$\Sigma F_y = N - mg\cos\theta = 0$$
$$N = mg\cos\theta$$
$$= 35\text{ kg} \times 9.8\text{ m s}^{-2} \times \cos 30°$$
$$= 297.05\text{ N}$$

$$\Sigma F_x = -f_k + mg\sin\theta = ma$$
$$\implies a = \frac{-f_k + mg\sin\theta}{m}$$
$$= \frac{-\mu_k N + mg\sin\theta}{m}$$
$$= \frac{-0.40 \times 297.05\text{ N} + 35\text{ kg} \times 9.8\text{ m s}^{-2} \times \sin 30°}{35\text{ kg}}$$
$$= 1.5\text{ m s}^{-2}$$

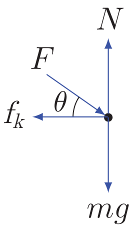
1 mark – Calculates the correct normal force

2 marks – Calculates the correct acceleration

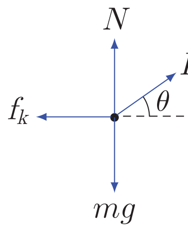
9. A bin is to be wheeled across a yard. This can be done by either pushing the bin from behind or pulling the bin from in front as shown in the photos below. **4**



By analysing the normal and frictional forces involved in each case, explain whether it is more efficient to push or pull the bin. Include relevant free body diagrams in your answer.



Case 1: Pushing



Case 2: Pulling

When the bin is pushed, the vertical component of the applied force acts down on the ground. This increases the net force pushing down on the ground, leading to a greater normal force. As the frictional force is proportional to the normal force, this results in a larger frictional force, which makes it more difficult for the bin to be moved.

$$
\begin{aligned}
\Sigma F_y &= N - mg - F\sin\theta = 0 \\
\implies N &= mg + F\sin\theta \\
f_{k,\,push} &= \mu_k N \\
&= \mu_k(mg + F\sin\theta)
\end{aligned}
$$

When the bin is pulled, the vertical component of the applied force is acting upwards. This reduces the net force on the ground, so the normal force and the frictional force are also reduced. This makes it more efficient to pull the bin.

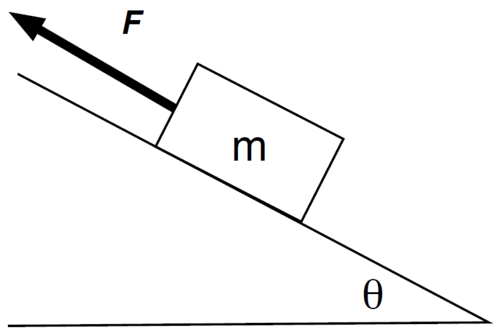
$$
\begin{aligned}
\Sigma F_y &= N - mg + F\sin\theta = 0 \\
\implies N &= mg - F\sin\theta \\
f_{k,\,pull} &= \mu_k N \\
&= \mu_k(mg - F\sin\theta) \\
&< f_{k,\,push}
\end{aligned}
$$

1 mark – Identifies that pulling the bin is more efficient

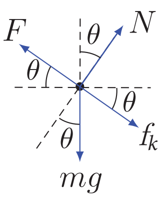
1 mark – Includes relevant free body diagrams for both cases

2 marks – Explains why pulling the bin is more efficient in terms of the relative normal and frictional forces in each case (can use either a worded explanation or equations)

10. A block with mass m lies on a plane inclined at an angle of θ. The coefficient of kinetic friction between the mass and the plane is μ_{k}. A force of magnitude F acting up the plane causes the mass to accelerate up the plane at a constant acceleration of a. The acceleration due to gravity is g. **4**



Show that $F = m(a + g(\sin\theta + \mu_{\text{k}}\cos\theta))$.



Resolving forces ($\nearrow^{+}$ and $^{+}\nwarrow$),

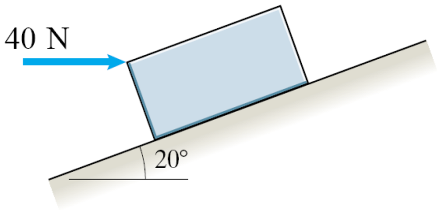
$$\Sigma F_y = N - mg\cos\theta = 0$$
$$N = mg\cos\theta$$

$$\Sigma F_x = F - f_k - mg\sin\theta = ma$$
$$\begin{aligned} \implies F &= ma + f_k + mg\sin\theta \\ &= ma + \mu_k N + mg\sin\theta \\ &= ma + \mu_k mg\cos\theta + mg\sin\theta \\ &= m(a + \mu_k g\cos\theta + g\sin\theta) \\ &= m(a + g(\sin\theta + \mu_{\text{k}}\cos\theta)) \end{aligned}$$

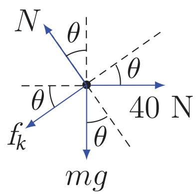
1 mark – Derives an expression for N by resolving forces

3 marks – Shows that $F = m(a + g(\sin\theta + \mu_{\text{k}}\cos\theta))$ with full working out

11. A 8 kg block is initially at rest on a plane inclined at 20° to the horizontal. The block is then pushed up the inclined plane with a horizontal force of 40 N. The coefficient of kinetic friction between the block and the surface is 0.1.



(a) Calculate the acceleration of the block when it is moving. **3**



Resolving forces ($^+\nwarrow$ and $\nearrow^+$),

$$\Sigma F_y = N - mg\cos\theta - (40\text{ N})\sin\theta = 0$$

$$\begin{aligned} N &= mg\cos\theta + (40\text{ N})\sin\theta \\ &= 8\text{ kg} \times 9.8\text{ m s}^{-2} \times \cos 20° + (40\text{ N})\sin 20° \\ &= 87.35\text{ N} \end{aligned}$$

$$\Sigma F_x = (40\text{ N})\cos\theta - f_k - mg\sin\theta = ma$$

$$\begin{aligned} \implies a &= \frac{(40\text{ N})\cos\theta - f_k - mg\sin\theta}{m} \\ &= \frac{(40\text{ N})\cos\theta - \mu_k N - mg\sin\theta}{m} \\ &= \frac{(40\text{ N})\cos 20° - 0.1(87.35\text{ N}) - (8(9.8)\text{ N})\sin 20°}{8\text{ kg}} \\ &= 0.25\text{ m s}^{-2} \end{aligned}$$

1 mark – Calculates the correct normal force

1 mark – Calculates the correct frictional force

1 mark – Calculates the correct coefficient of friction

(b) Calculate the speed of the block and the distance it has moved after 3 seconds. **2**

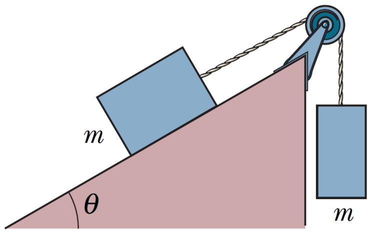
$$\begin{aligned} v &= u + at \\ &= 0 + 0.25 \text{ m s}^{-2} \times 3 \text{ s} \\ &= 0.75 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ (0.75 \text{ m s}^{-1})^2 &= 0^2 + 2(0.25 \text{ m s}^{-2})s \\ \implies s &= 1.13 \text{ m} \end{aligned}$$

1 mark – Calculates the correct speed

1 mark – Calculates the correct distance

12. Two blocks with equal mass (m) are connected via a light, inextensible string over a frictionless pulley. One of the masses slides on a plane inclined at an angle of θ to the horizontal. The coefficient of kinetic friction between this mass and the inclined plane is μ_k. When released, the mass on the right accelerates downwards with an acceleration of a. The acceleration due to gravity is g.



(a) Draw TWO free body diagrams showing all the forces acting on each of the blocks. **3**

3 marks – Draws two free body diagrams showing the forces acting on both blocks

(b) Show that $a = \frac{g}{2}(1 - \sin\theta - \mu_\mathrm{k}\cos\theta)$. **4**

Summing up forces on the right block ($\downarrow_+$),

$$\Sigma F_y = -T + mg = ma$$
$$\implies T = mg - ma$$

Resolving forces ($^+\nwarrow$ and $\nearrow^+$) on the left block,

$$\Sigma F_y = N - mg\cos\theta = 0$$
$$N = mg\cos\theta$$

$$\begin{aligned}\Sigma F_x = T - f_k - mg\sin\theta &= ma\\ \implies a &= \frac{T - f_k - mg\sin\theta}{m}\\ &= \frac{T - \mu_k N - mg\sin\theta}{m}\\ &= \frac{mg - ma - \mu_k mg\cos\theta - mg\sin\theta}{m}\\ &= g - a - \mu_k g\cos\theta - g\sin\theta\\ \implies 2a &= g - \mu_k g\cos\theta - g\sin\theta\\ &= g(1 - \sin\theta - \mu_k\cos\theta)\\ \implies a &= \frac{g}{2}(1 - \sin\theta - \mu_\mathrm{k}\cos\theta)\end{aligned}$$

1 mark – Derives an expression for T by applying Newton's 2nd law on the right block

1 mark – Derives an expression for N

2 marks – Shows that $a = \frac{g}{2}(1 - \sin\theta - \mu_k \cos\theta)$ with full working out